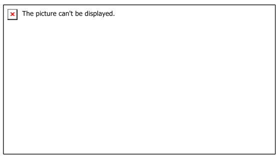

UIN: [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]   Sub. Code : 19KBMAT21

Khaja Bandanawaz University
Faculty of Engineering and Technology
Second Semester B. E. Degree Examination
Sub : Advanced Calculus and Laplace Transform

Time : 3 Hrs   Max. Marks : 100

Section – A

I. Answer any TEN Questions from the following : (02 Marks Each)

1. Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$
2. Obtain the complementary function of $y'' + 9y = 0$.
3. Solve $(D^2 + 6D + 9)y = 0$
4. Obtain the complementary function of $x^2y'' - 3xy' + 4y = (1 + x)$
5. Write the general solutuion of $(2x - 1)^2y'' + (2x - 1)y' - 2y = 0$.
6. Obtain the general solution of $y = px + p^2$.
7. Form the partial differential equation by eliminating the arbitrary constants from the equation $z = ax + by + ab$.
8. Form the partial differential equation by eliminating the arbitrary functions from the equation $z = f(xy)$.
9. Obtain the general solution of the equation $p = \sin(y - xp)$.
10. Evaluate the value of $\Gamma(-\frac{5}{2})$.
11. Compute the value of $\mathrm{B}(3, \frac{5}{2})$.
12. Evaluate $\int_1^2 \int_0^x dy\, dx$.
13. Find the Laplace transform of $1 + 2t^3 - 4e^{3t}$.
14. Find the inverse Laplace Transform of $\frac{2s}{s^2+9}$.
15. Obtain the Laplace Transform of $\frac{1}{s+1} - \frac{2}{s-1} + \frac{3}{s^4}$.

Section – B

II. Answer any FIVE full questions from the following : (08 Marks Each)

1. (a) Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$
   (b) Find general solution of $(D^2 + 7D + 17)y = \cosh x$, given that the complementary function is $y = Ae^{-3x} + Be^{-4x}$.
2. (a) Using the method of undetermined coefficients solve $y'' - 3y' + 2y = x^2 + e^x$ given that the Complementary function is $y = Ae^x + Be^{2x}$
   (b) Obtain the Particular Integral of $(D^3 + 6D^2 + 11D + 6)y = e^x + 1$, given that the complementary function is $y = Ae^{-x} + Be^{-2x} + Ce^{-3x}$.

3. Solve $x^3y''' + 3x^2y'' + xy' + 8y = 65\cos\log x$.
4. (a) Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ by solving for p.
   (b) Solve $y = 2px + y^2p^3$ by solving for x.
5. (a) Form the partial differential equation by eliminating the arbitrary constants from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
   (b) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \sin(3 + 2y)$.
6. Obtain the various possible solutions of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ by the mehod of separation of variables.
7. (a) Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y} xyz\, dz\, dy\, dx$.
   (b) Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\, dy\, dx$ by changing the order of integration.
8. By employing the convolution theorem, evaluate the inverse Laplace Transform of $\frac{s}{(s^2+a^2)^2}$.

Section – C

III. Answer any FOUR full questions from the following : (10 Marks Each)

1. (a) Using the method of variation of parameters solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$
   (b) Find the general solution of $(D^2 - 2D + 5)y = e^{2x}\sin x$.
2. (a) Solve $(px - y)(py + x) = 2p$, by reducing into Clairauit's form taking the substitution $X = x, Y = y$.
   (b) Solve $(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 2\sin\log(1 + x)$.
3. (a) Derive one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$.
   (b) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.
4. (a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
   (b) Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\, d\theta$ x $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \pi$.
5. (a) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\, dy$ by changing to polar coordinates.
   (b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.
6. (a) If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$, $f(t + 2a) = f(t)$, then show that L$\{f(t)\} = \frac{1}{s^2}\tanh\frac{as}{2}$.

   (b) Express $f(t) = \begin{cases} \sin t, & 0 < t \le \pi/2 \\ \cos t, & t > \pi/2 \end{cases}$ in terms of the unit step function and hence find its Laplace transform.

7 (a) Solve by using Laplace transform method :

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, \qquad y(0) = 0, y'(0) = 0$$

   (b) Obtain the inverse Laplace Transform of $\frac{s+5}{s^2-6s+13}$.