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Khaja Bandanawaz University

Faculty of Engineering and Technology

Second Semester B. E. Degree Examination

Sub : Differential Calculus and Linear Algebra

Time : 3 Hrs Max. Marks : 100

Section A

I. Answer any TEN Questions from the following : (02 Marks Each)

1. If $\vec{f} = (x+y+1)i + j - (x+y)k$, show that $\vec{f}.\,curl\,\vec{f} = 0$.
2. Show that $\vec{f} = (z+\sin y)i + (x\,cos\,y - z)j + (x-y)k$ is irrotational.
3. If $x = u(1-v), y = uv$, find $\frac{\partial(x,y)}{\partial(u,v)}$.
4. Find div $\vec{F}$ where $\vec{F} = grad(x^3+y^3+z^3-3xyz)$.
5. Evaluate $\lim\limits_{x\to 1} x^{1/(1-x)}$.
6. Evaluate $\int_0^{2\pi} cos^4 x\,dx$.
7. Evaluate $\int_0^{\pi/2} sin^6 x\,cos^5 x\,dx$.
8. Check whether the differential equation $3x(xy-2)dx + (x^3+2y)dy = 0$ is exact differential equation.
9. Define rank of a matrix. Also write the elementary row transformations.
10. Show that for the polar curve $r = ae^{\theta\cot\alpha}$, where α is a constant, the radius vector is inclined at a constant angle to the tangent, at every point.
11. Find the eigen values of the matrix $\begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$.
12. Solve the system of equations: $x+2y+3z=0, 3x+4y+4z=0, 7x+10y+12z=0$.
13. If $u = x^y$, obtain $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.
14. Write down the expressions for radius of curvature in cartesian form and polar form.
15. State Taylor's theorem for a function of one variable.

Section B

II. Answer any FIVE full questions from the following : (08 Marks Each)

1. (a) With usual notation, prove that $\tan\emptyset = r\frac{d\theta}{dr}$.

   (b) Prove that $\tan^{-1}x = \tan^{-1}\frac{\pi}{4} + \frac{(x-\frac{\pi}{4})}{1+\frac{\pi^2}{16}} - \frac{\pi\left(x-\frac{\pi}{4}\right)^2}{4\left(1+\frac{\pi^2}{16}\right)^2} + \cdots$
2. (a) Obtain the pedal equation of the curve $r^m\cos m\theta = a^m$.

   (b) Find the angle between the curves $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$.

3. (a) If $u = \tan^{-1}\frac{y}{x}$ where $x = e^t - e^{-t}, y = e^t + e^{-t}$, find $\frac{du}{dt}$.
   (b)Find the maximum value of the function $f(x, y) = x^3y^2(1 - x - y), x, y \neq 0$.
4. (a) If $u = x + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at the point (1, -1, 0).
   (b) Evaluate $\lim_{x\to\pi/2}(\cos x)^{\frac{\pi}{2}-x}$
5. (a) Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at the point (1, -2, -1) along the vector $\vec{a} = 2i - j - 2k$.
   (b) If C is the boundary of the triangle with vertices at P(1, 0, 0), Q(0, 2, 0) and R(0, 0, 3), evaluate $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$ by using Stokes's theorem.

6. (a) A body originally at 80℃ cools down to 60℃ in 20 minutes in the surroundings of temperature 40℃, find the temperature of the body after 40 minutes from the original instant.
   (b) Evaluate: $\int_0^{2a} x\sqrt{2ax - x^2}\, dx$.
7. (a) Obtain the Reduction formula for $\int_0^{\pi/2} \sin^n x\ dx$.
   (b) Solve : $\frac{dy}{dx} - \frac{2}{x}y = \frac{y^2}{x^3}$.
8. (a) Reduce the matrix $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ to diagonal form.
   (b) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Section C

III. Answer any FOUR full questions from the following : (10 Marks Each)

1. (a) Find the orthogonal trajectories of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$, where λ is a parameter.
   (b) Solve : $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$.

2. (a) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational.
   (b) Using Green's theorem, find the area enclosed between the parabolas $x^2 = 4ay\ and\ y^2 = 4ax$.
3. (a) Find the radius of curvature of $x^{2/3} + y^{2/3} = a^{2/3}$ at any point (x , y).
   (b) Prove that $\log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots$

4. (a) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

   (b) If $z(x + y) = x^2 + y^2$, show that $(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y})^2 = 4(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y})$.

5. (a) Verify the Divergence theorem for $\vec{f} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

   (b) If C is the boundary of the triangle with vertices at P(1, 0, 0), Q(0, 2, 0) and R(0, 0, 3), evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$ by using Stokes's theorem.

6. (a) Using Power method find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ with $X^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Carry out 5 iterations.

   (b) Solve Gauss-Seidel method : $2x + y + 6z = 9; 8x + 3y + 2z = 13; x + 5y + z = 7.$

   Carry out 5 iterations to obtain solution correct to 4 decimal places.

7. (a) Find the values of a and b for which the equations:

   $x + y + z = 3, x + 2y + 2z = 6, \; x + ay + 3z = b$

   has (i) no solution, (ii) a unique solution, (iii) infinitely many solutions.

   (b) Solve by Gauss–Jordan method :

   $2x + y + z = 10; \; 3x + 2y + 3z = 18; \; x + 4y + 9z = 16.$

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