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UIN: Sub. Code : 19KBMAT11

Khaja Bandanawaz University

Faculty of Engineering and Technology

First Semester B. E. Degree Examination

Sub : Differential Calculus and Linear Algebra

Model question paper -2

Time : 3 Hrs Max. Marks : 100

Section A

I. Answer any TEN Questions from the following : (02 Marks Each)

1. Write the expression for the angle between the radius vector and tangent for a polar curve $r = f(\theta)$. Also define pedal equation.
2. Find the angle between the curves $r = 2\sin\theta,\ \ r = 2\cos\theta.$
3. Obtain the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \pi/2$.
4. If $u = e^{xyz}$, evaluate $\frac{\partial^2 u}{\partial y \partial z}$.
5. If $x = r\,cos\,\theta, y = r\,sin\,\theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
6. Evaluate $\lim_{x\to 0}(\cos x)^{1/x^2}$.
7. Evaluate :$\int_0^{\pi/6} \sin^2 6x\ \cos^4 6x\ dx$ using Reduction formula.
8. Show that $((3x^2y^2 + x^2)dx + (2x^3y + y^2)dy = 0$ is exact differential equation.
9. Evaluate $\int_0^{\pi/8} cos^5 4x\ dx$.
10. When is a vector $\vec{F}$ said to be solenoidal and when is it irrotational ?
11. Define directional derivative. Where is it maximum and what is its maximum value ?
12. If If $\vec{F} = \nabla(xy^3z^2)$ find div $\vec{F}$ at the point (1, -1, 1).
13. Write the diagonal form of $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$.
14. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.
15. Write down the condition for consistency for a non-homogeneous system of linear equations. Also write the nature of solution depending on the value of rank of the matrix.

Section B

II. Answer any FIVE full questions from the following : (08 Marks Each)

1. (a)Derive the expression for the length of the perpendicular from the pole on to the tangent for the polar curve $r = f(\theta)$.

(b) Find the angle between the radius vector and tangent for the curve $r = a(1 + \cos\theta)$ and also find the slope of the tangent at $\theta = \pi/3$.

2. (a) Find the angle between the curves $r^2 \sin 2\theta = 4\ and\ r^2 = 16 \sin 2\theta$.

   (b) Obtain the pedal equation of the curve $r^n = a(1 + \cos n\theta)$.

3. (a) If $x = r\sin\theta\cos\emptyset$, $y = r\sin\theta\sin\emptyset, z = r\cos\theta$, show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2 \sin\theta$.

   (b) If $u = (x^2 + y^2 + z^2)^{-1/2}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

4. (a) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

   (b) Evaluate : $\lim_{x\to\pi/2} (\sin x)^{\tan(x)}$.

5. Show that $\vec{F} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is a conservative force field. Find its scalar potential.

6. (a) Evaluate : $\int_0^\infty \frac{x^2}{(1+x^2)^{7/2}}\, dx$

   (b) Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal.

7. (a) Obtain the reduction formula for $\int cos^n x\, dx$.

   (b) Solve : $\frac{dy}{dx} + y\tan x = y^3 \sec x$.

8. (a) Find the values of λ for which the system of equations : $x + y + z = 1;\ x + 2y + 4z = \lambda;$

   $x + 4y + 10z = \lambda^2$ has a solution. Solve it in each case.

   (b) Find the rank of the matrix $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$.

Section C

III. Answer any FOUR full questions from the following : (10 Marks Each)

1. (a) Obtain the Taylor's series expansion of $f(x) = \log(\cos x)$ about the point $x = \pi/3$ upto the fourth degree term.

   (b) Find the radius of curvature of the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where it cuts the x-axis.

2. (a) If $z = xy^2 + x^2y$ where $x = at^2, y = 2at$, find $\frac{dz}{dt}$. Also verify the result by direct substitution.

   (b) If $z = f(u, v)$, where $u = x^2 - y^2$ and $v = 2xy$, prove that $(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = 4(x^2 + y^2)[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2]$.

3. (a) Find the directional derivative of $\emptyset = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ along $2i - 3j + 6k$.

   (b) If $\vec{F} = \nabla(xy^3z^2)\ find\ div\ \vec{F}\ and\ curl\ \vec{F}\ at\ the\ point\ (1,-1,1)$.

4. (a)Using Green's theorem evaluate $\int_C (y - \sin x)dx + \cos x\, dy$, where C is the triangle in

the xy plane formed by the lines $y = 0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$.

(b) Verify Stoke's theorem for $\overrightarrow{f} = yi + zj + xk,$ for the upper part of the sphere

$x^2 + y^2 + z^2 = a^2$.

5. (a) Solve : $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

(b) If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes.

6. (a) Using Power method find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ with $X^{(0)} = \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix}$. Carry out 5 iterations.

(b) Solve by Gauss-Seidel method : $28x + 4y - z = 32; x + 3y + 10z = 24; 2x + 17y + 4z = 35$. Carry out 5 iterations to obtain solution correct to 4 decimal places.

7. (a) Solve by Gauss–Jordan method : $x + y + z = 9;\ 2x + y - z = 0;\ 2x + 5y + 7z = 52$.

(b) Diagonalize the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and hence find A^4.

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